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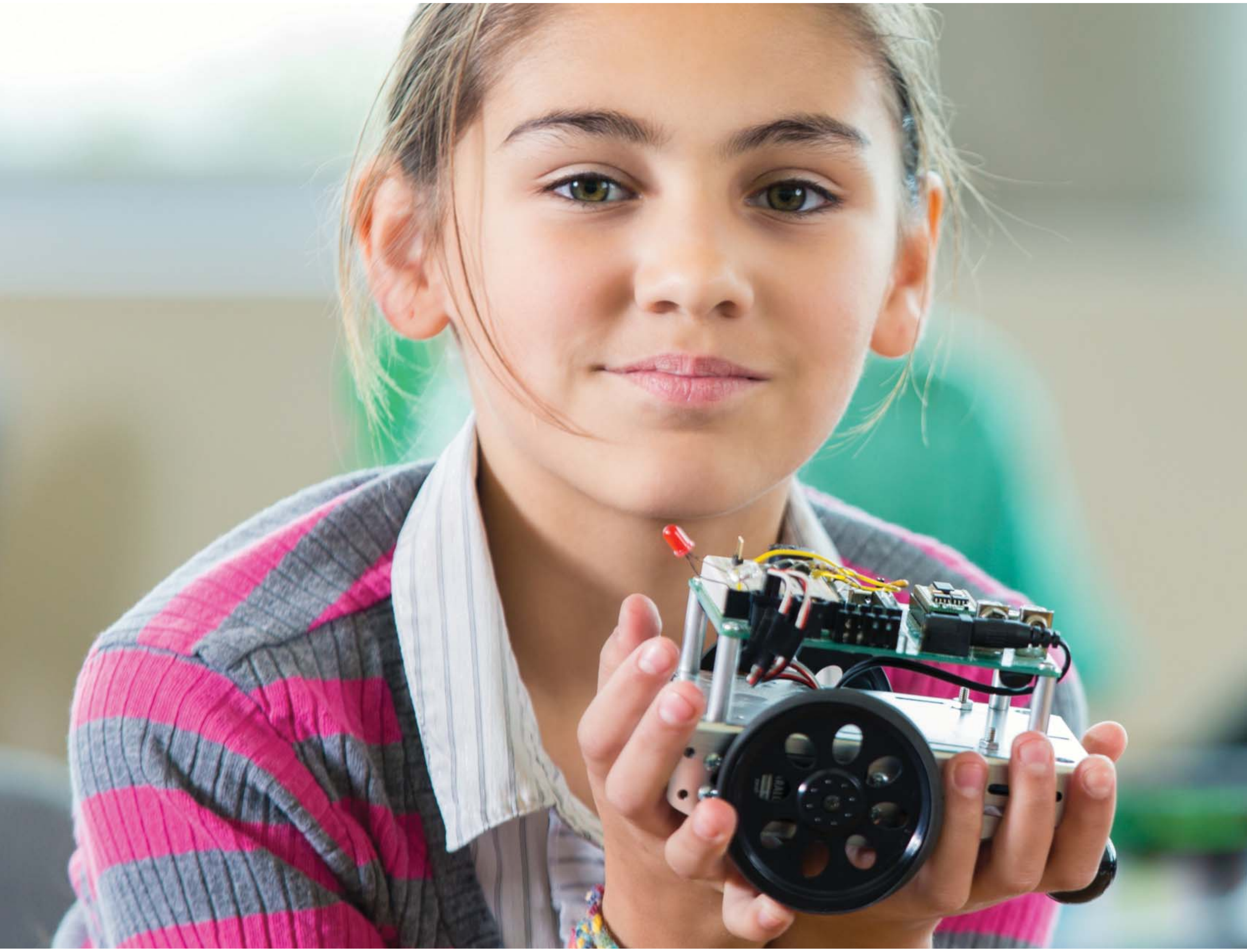

idea packet

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Transformations are Easier if the Force is with You

Transformations are Easier if the Force is with You

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Goals & Objectives

The primary goal of this project is to lead students through abstract concepts associated in algebraic functions. This project has inspired my students to discover the rules for rigid transformations of functions. The inquiry based lessons used the Ti-84C, a color graphing calculator, and emulator software projected on an interactive whiteboard, to develop their own rules, rather than being taught these rules by the instructor. This method of instruction empowers the students to accept responsibility for their own learning.

In addition, this linked readily to the semester project where students identified ten types of functions with pictures representing their graphs. An example that many students identified with are the Golden Arches of McDonald’s which is represented by a pair of parabola with leading coefficients of negative integers.

These projects, when completed in tandem, allowed students to connect functions and their graphs to their everyday life. So many of our students question how complicated Algebra concepts connect to their daily life; asking, “When will I use this after High School?

Course Outline

- Transformations can be categorized into two types. Those that maintain congruence. Functions that shift vertically, horizontally or a combination of both (i.e. rigid transformations). Additionally, there are other transformations that are not rigid such as a vertical shift or compression. These do not maintain congruency.

- The Behaviors of functions depend on two factors; the degree of the polynomial and the leading coefficient.

- The Axis of Symmetry can be calculated using the formula $-b/2a$ when the polynomial is written in standard form.

- The Vertex can be found when the polynomial is written in either standard or vertex form.

- The x and y Intercepts can be found algebraically and/or graphically. In this project students learn how to find them graphically first.

Standards / Objective

MAFS.K12.MP.5.1

Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MAFS.K12.MP.4.1

Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MAFS.912.G-CO.1.2

Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

MAFS.912.A-APR.2.3

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Lesson Plan

- **Learning Objectives: What will students know and be able to do as a result of this lesson?**

  The student will be able to:

  - Identify differences and similarities between a function and its transformation.
  - Identify a graph of a function given a graph or a table of a transformation and the type of transformation that is represented.
  - Graph by applying a given transformation to a function.
  - Identify ordered pairs of a transformed graph.
  - Complete a table for a transformed function.
  - Recognize even and odd functions from their graphs and equations

- **Prior Knowledge: What prior knowledge should students have for this lesson?**

  Students should know and be able to:

  - In Grade 8, students compare tables, graphs, expressions and equations of linear relationships.
  - Understanding slope as a rate of change, as well as working with integral exponents, are important elements of the Grade 8 curriculum.
  - Students should be able to use graphing calculators or other graphing utilities to generate tables of values, graph, or solve a variety of functions.

- **Guiding Questions: What are the guiding questions for this lesson?**
  - What is the relationship between the transformations in linear functions learned in 8th grade and non-linear functions such as quadratic and exponential functions? Students will recognize how the form of the expression or equation can provide information about its graph (vertex, minimum, maximum, etc.).
  - How do numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution method differ and produce precise solutions that can be represented graphically or numerically?

- **Introduction: How will the teacher inform students of the intent of the lesson? How will students understand or develop an investigable question?**

  Video Opening: The teacher may wish to show the video "Scream and Shout Polynomial Functions" by YouTube user My Vids. (https://www.youtube.com/watch?v=2F9WOwAnSkI). This video is a real attention getter, explaining ending behaviors of several functions.

  If you choose not to use or cannot use this video, you may want to use examples of parent functions and their transformations. The teacher should present these graphs using either graphing calculators or graphing utilities such as www.demos.com .

- **Investigate: What will the teacher do to give students an opportunity to develop, try, revise, and implement their own methods to gather data?**

  Students will use graphing calculators or other graphing utilities to complete a chart that includes parent functions and sample transformations. See the attached worksheet.

  It should not take long for students to see the connections between the parent functions and their transformations. (see Analyze)

- **Analyze: How will the teacher help students determine a way to represent, analyze, and interpret the data they collect?**

  Students will use the activity provided to determine the rules for transformations. This is an inquiry based lesson where the teacher is a "Guide on the Side" rather than the "Sage on the Stage". The teacher's role is to circulate from each learning group, asking questions and providing feedback.

- **Closure: What will the teacher do to bring the lesson to a close? How will the students make sense of the investigation?**

  Students will complete an Exit Ticket consisting of parent functions and their transformation and ask each student to explain their response either by using their calculator screen or referencing the rules that they discovered.

  As an alternative to the Exit Ticket, each student could revisit their group discussion and write a detailed explanation of their conclusion and how they reached an agreement.

- **Summative Assessment**

  Students will complete and submit an Exit Ticket that states: Other than the examples discussed in class, create two examples of a parent function and a transformation.

- **Formative Assessment**

  As the lesson is taught, the teacher will circulate throughout the room, monitor students, ask focus questions, and provide verbal praise and constructive feedback. The teacher can gauge student mastery by monitoring discussion while students use graphing calculators or graphing utilities to complete a chart that includes parent functions and sample transformations. Students explain the transformations based on their observations.

- **Feedback to Students**

  Use the Four Levels of Mastery; see the video [The 4 Levels of Mastery] from YouTube user The Happiness Guy.

  1. Getting Started: Common Misconception → The student does not understand how to use the graphing calculator or graphing utilities.
     Possible questions:

     - How do we use the tools provided to graph parent functions and their transformations?
     - What rule can we use to explain the effect of the transformations on the parent function?

2. On the Way: Common Misconception → The student understands how to use graphing tools but is hesitant to draw conclusions on their own.
   Possible questions:

   - What is the obvious rule explaining the vertical transformations?
   - How does differ when you want a horizontal transformation?

3. Almost There: → The student understands how to use graphing tools but may be confused when combining vertical and horizontal transformations. In addition, the student understands how to use the vertex form to impose the desired transformations.
   Possible questions:

   - Using the rules that you have created and the vertex form of a function; How do I transform a function using a new vertex (h,k)

4. Got it→ The student understands how to use graphing tools when combining vertical and horizontal transformations. In addition, the student understands how to use the vertex form to impose the desired transformations and explain the projected transformation.

ACCOMMODATIONS& RECOMMENDATIONS

- **Accommodations:**

  Cooperative learning assists students at different cognitive levels attain mastery regardless of their individual abilities. The teacher should develop groups of varying levels. If this class uses an inclusion model, the inclusion teacher may ask probing questions to the group, however, deeper understanding occurs when students draw their own conclusions, thus developing their own rule.

- **Extensions:**
  - How can I use skills such as completing the square to convert a function in standard to vertex form?

- **Suggested Technology:** Computer for Presenter, Interactive Whiteboard, LCD Projector, Microsoft Office, Graphing Calculator and Emulator Software.

- **Special Materials Needed:**

  The teacher should prepare for this lesson by downloading the activity worksheet with the chart to be completed by students. Each student will also need a graphing calculator or access to graphing tools. The teacher may also choose to prepare presentation tools created in software like Smart Notebook or Active Inspire.

  Special care should be made to allow each group to draw their own conclusion. Remember that an inquiry-based lesson should allow students to develop the rules for mathematical relationships rather than learning the rule or relationship from their teacher. This should develop a deeper "Depth of Knowledge" for the learner.

- **Further Recommendations:**

- Special care should be taken to allow enough time for each group to develop their own rules for this relationship. In addition, the groups should consist of learners with different cognitive abilities.
- Each student will need an activity worksheet.

Resource List:

• Student Worksheets

- Directions: Using the Ti-84C or another graphing Utility, describe the transformation has on the parent function.

- Linear Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
Y= x	Y= x + 2	
Y= x	Y= x - 4	
Y= x	Y= -3x	
Y= x	Y= (1/2) x	
Y= x	Y= 2x - 2	

- Quadratic Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
Y= x^2	Y= x^2 + 2	
Y= x^2	Y= x^2 - 4	
Y= x^2	Y= $-x^2$	
Y= x^2	Y= (1/2) x^2	
Y= x^2	Y= $2x^2$ - 2	

- Quadratic Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
$Y= x^2$	$Y= (x + 2)^2$	
$Y= x^2$	$Y= (x - 4)^2$	
$Y= x^2$	$Y= -(x + 2)^2 + 2$	
$Y= x^2$	$Y= (x - 2)^2 - 2$	

- Cubic Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
$Y= x^3$	$Y= x^3 + 2$	
$Y= x^3$	$Y= x^3 - 4$	
$Y= x^3$	$Y= -3x^3$	
$Y= x^3$	$Y= (1/2)\ x^3$	
$Y= x^3$	$Y= 2x^3 - 2$	

- Cubic Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
$Y= x^3$	$Y= (x + 2)^3$	
$Y= x^3$	$Y= (x - 4)^3$	
$Y= x^3$	$Y= -(x + 2)^3 + 2$	
$Y= x^3$	$Y= (x - 2)^3 - 2$	

- Absolute Value Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
Y= Abs (x)	Y= Abs (x) + 2	
Y= Abs (x)	Y= Abs (x) - 4	
Y= Abs (x)	Y= -3 Abs (x)	
Y= Abs (x)	Y= (1/2) Abs (x)	

- Absolute Value Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
Y= Abs (x)	Y= Abs (x + 2)	
Y= Abs (x)	Y= Abs (x – 4)	
Y= Abs (x)	Y= -Abs (3x)	
Y= Abs (x)	Y= Abs (½)x	

Resource List:

• Student Worksheets (Sample Answers)

Directions: Using the Ti-84C or another graphing Utility, describe the transformation has on the parent function.

Linear Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
Y= x	Y= x + 2	The Function translates 2 units up with the same slope.
Y= x	Y= x - 4	The Function translates 4 units down with the same slope.
Y= x	Y= -3x	The Function is reflected over the y-axis with a steeper slope with the same y intercept.
Y= x	Y= (1/2) x	The Function is similar but with a flatter slope with the same y intercept.
Y= x	Y= 2x - 2	The Function is reflected over the y axis with a steeper slope with the y intercept at (0,-2).

Quadratic Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
Y= x^2	Y= x^2 + 2	The Function opens up and is translated 2 units up.
Y= x^2	Y= x^2 - 4	The Function opens up and is translated 4 units down.
Y= x^2	Y= $-x^2$	The Function opens down with the same point as the maximum and minimum.
Y= x^2	Y= (1/2) x^2	The Function opens up with the same point as the minimum but with a vertical stretch.
Y= x^2	Y= $2x^2$ - 2	The Function opens up with the same point as the minimum but with a vertical stretch and a new minimum at (0,-2)

Quadratic Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
Y= x^2	Y= $(x + 2)^2$	The Function opens up like the parent function but is translated to the left 2 spaces.
Y= x^2	Y= $(x - 4)^2$	The Function opens up like the parent function but is translated to the right 4 spaces.
Y= x^2	Y= $-(x + 2)^2 + 2$	The Function opens down shifts left 2 spaces and is translated up 2 spaces.
Y= x^2	Y= $(x - 2)^2 - 2$	The Function opens up shifts right 2 spaces and is translated down 2 spaces.

Cubic Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
Y= x^3	Y= $x^3 + 2$	The ending behaviors of the Function are down and up like the parent function and is translated up 2 spaces.
Y= x^3	Y= $x^3 - 4$	The ending behaviors of the Function are down and up like the parent function and is translated down 4 spaces.
Y= x^3	Y= $-3x^3$	The ending behaviors of the Function are up and down, unlike the parent function. The new function has a new vertical shift.
Y= x^3	Y= $(1/2)\ x^3$	The ending behaviors of the Function are down and up, like the parent function. The new function has a new vertical compression with the same y intercept.
Y= x^3	Y= $2x^3 - 2$	The ending behaviors of the Function are down and up, like the parent function. The new function has a new vertical shift and translated 2 units down.

Cubic Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
Y= x^3	Y= $(x + 2)^3$	The ending behavior of the Function is down and up like the parent function but is translated to the left 2 spaces.
Y= x^3	Y= $(x - 4)^3$	The ending behavior of the Function is down and up like the parent function but is translated to the right 4 spaces.
Y= x^3	Y= $-(x + 2)^3 + 2$	The ending behavior of the Function is up and down unlike the parent function but is translated to the left 2 spaces and up 2 spaces.
Y= x^3	Y= $(x - 2)^3 - 2$	The ending behavior of the Function is down and up like the parent function but is translated to the right 2 spaces and down 2 spaces.

Absolute Value Functions

Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
Y= Abs (x)	Y= Abs (x) + 2	The Function opens up in the shape of a V, and is translated 2 units up.
Y= Abs (x)	Y= Abs (x) - 4	The Function opens up in the shape of a V, and is translated 4 units down.
Y= Abs (x)	Y= -3 Abs (x)	The Function opens down in the shape of a V, reflected over the x axis with a vertical shift.
Y= Abs (x)	Y= (1/2) Abs (x)	The Function opens up in the shape of a V, vertical compression, flattening out the graph.

Absolute Value Functions

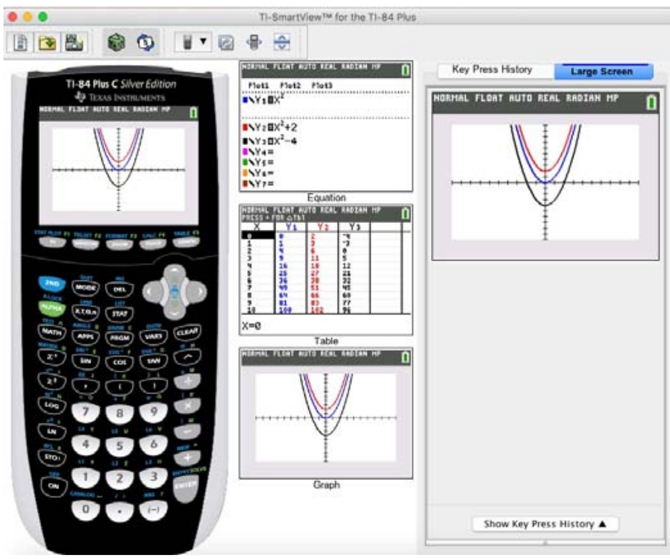
Parent Function	Transformation	In your own words, describe the effect of the transformation on the Parent Function.
Y= Abs (x)	Y= Abs (x + 2)	The Function opens up in the shape of a V, and is translated 2 units to the left.
Y= Abs (x)	Y= Abs (x – 4)	The Function opens up in the shape of a V, and is translated 4 units to the right.
Y= Abs (x)	Y= -Abs (3x)	The Function opens down in the shape of a V, reflected over the x axis with a vertical shift.
Y= Abs (x)	Y= Abs (½)x	The Function opens up in the shape of a V, with a vertical compression, flattening out the graph.

Possible Adaptations

- Emulator Software
- Graphing Utilities

This presentation was created using Ti-84c graphing calculators and Ti-SmartView Emulator software. Although the target audience is Algebra 1 & 2, preliminary grades from 5th to 8th may also use some of the applications from this project.

Included in this section are screenshots from several resources. This screen shot is from the emulator software: Ti SmartView.

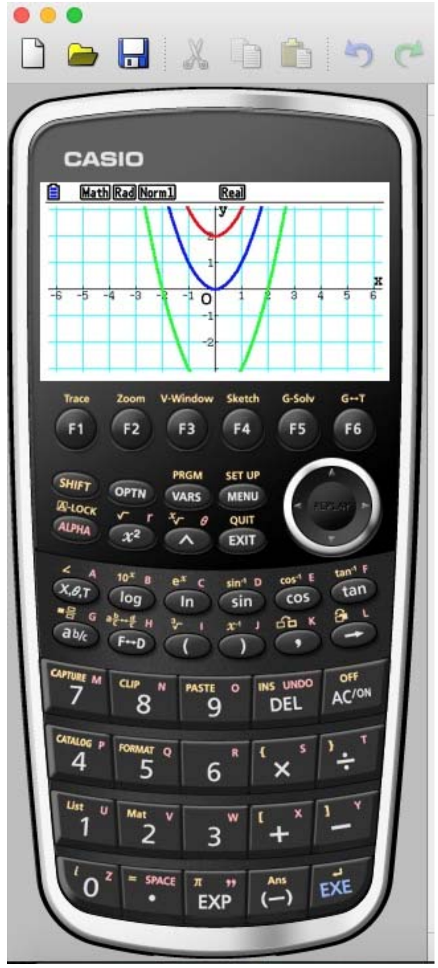


You can buy the software for $138.51 from D&H Distributing along with several other distributors. You may also search https://education.ti.com/en/us/purchase/purchase_dealer for other possible retailers. Additionally, this is an exceptional tool that allows the student to follow along with the teacher, step by step. This is a product that teachers with access to school owned Ti graphing calculators should consider to make graphing presentations/lessons more efficient and easier for students to follow. Without it, get used to a roomful of students all needing individual help.

In addition, Texas Instruments has developed an app for iPads; Ti-Nspire (29.99 per download) If you are interested in this software you may want to download the 30 day trial before making this purchase.

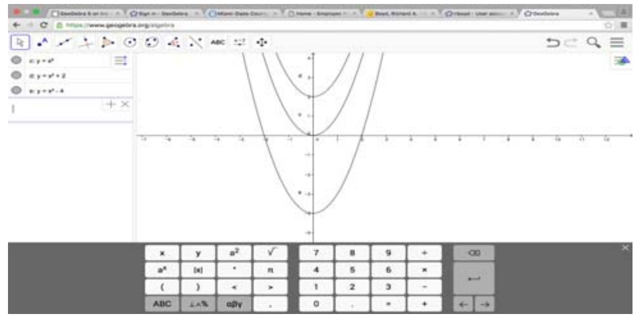
Another graphing calculator that may be available at your school is the Casio Prizm (fx-CG). This is a color version, but they do market a less expensive product that is not a color version. This emulator software at one time was available for free to teachers after completing several lessons.
https://edu.casio.com/softwarelicense/index.php#col2

The following is an example of a screen shot from the Casio fx CG emulator software.

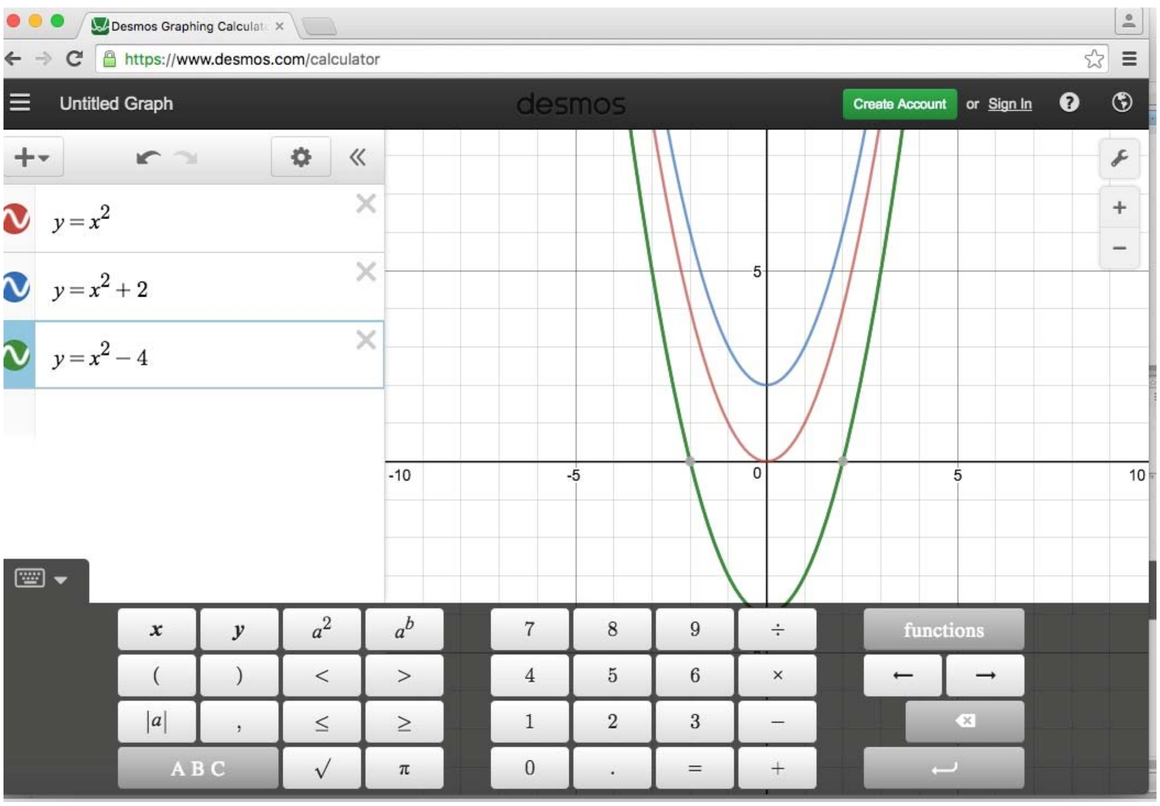


Other graphing utilities that can be used on a computer, laptop, or tablet (BYOD) include but are not limited to Desmos https://www.desmos.com/ or GeoGegra https://www.geogebra.org/algebra. The interfaces of these two products are very similar. These web based programs/applications can serve as a bridge for the teacher that is building their supply of graphing calculators a few at a time.

The following is a screen shot from GeoGebra.



The following is an example of a screen shot from Desmos.



Any or all of these graphing programs or utilities can be used to adapt this project to fit the needs of your classroom. I found the Ed Fund to very helpful in building my class supply of Ti-84c calculators and emulator software.

THE EDUCATION FUND CELEBRATING 30 YEARS

APPLY FOR AN IMPACT II ADAPTER GRANT!

M-DCPS teachers, media specialists, counselors or assistant principals may request funds to implement an IMPACT II idea, teaching strategy or project from the Idea EXPO workshops and/or curriculum ideas profiled annually in the ***Ideas with IMPACT*** catalogs from 1990 to the current year, 2016-17. Most catalogs can be viewed at The Education Fund website at www.educationfund.org.

- Open to all K-12 M-DCPS teachers, counselors, media specialists
- Quick and easy reporting requirements
- Grants range from $150 - $400
- Grant recipients recognized at an Awards Reception

To apply, you must contact the teacher who developed the idea before submitting your application. Contact can be made by attending a workshop given by the disseminator, communicating via email or telephone, by visiting the disseminator in their classroom, or by having the disseminator visit your classroom.

Project funds are to be spent within the current school year or an extension may be requested. An expense report with receipts is required by Friday, May 5, 2017.

APPLICATION DEADLINE:
Monday, December 12, 2016

Apply online at www.educationfund.org

For more information, contact:

Edwina Lau, Program Director
305.558.4544, ext. 113
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